Low-frequency electrostatic waves in a hot magnetized dusty plasma

M. R. Amin
Department of Physics, Jahangirnagar University, Savar, Dhaka 1342, Bangladesh
(Received 21 February 1996)

A gyrokinetic study of low-frequency—lower than the dust cyclotron frequency—electrostatic modes in a magnetized dusty plasma has been presented. It is found that the inequalities of charge and number densities of electrons, ions, and dust particles, and the finite-Larmor-radius thermal kinetic effects of the mobile charged dust grains, introduce the existence of low-frequency electrostatic eigenmodes in a three-component hot magnetized dusty plasma. The relevance of our investigation to space and astrophysical as well as laboratory experiments for dust-particulate attraction is pointed out. [S1063-651X(96)50409-5]

PACS number(s): 52.35.Fp, 52.25.Mq, 52.35.Mw, 52.35.Ra

Dusty plasmas [1–20] are finding numerous applications in systems ranging from space and astrophysical situations and planetary atmospheres to laboratory and modern technologies as well as in macroscopic Coulomb crystal formation. Electrons, ions, and electrically charged extremely heavy dust grains are the constituents of a dusty plasma. The dust grains are usually negatively charged, with a large number of electrons on each dust grain ($Z_d \sim 10^3 - 10^4$), they are of micrometer and submicrometer size, and they have a mass that is much larger than the positive ion mass ($m_d/m_p \sim 10^6 - 10^{18}$).

There are two important features that distinguish dusty plasmas from the usual multiple-component plasmas, which have either more than one kind of ion species or electron species with different temperatures. First, the presence of the very massive, charged dust grains introduces new time and space scales in the plasma behavior, leading to new waves and instabilities [21–24]. Second, the charges on the dust particles can vary, owing either to the wave-motion-induced electron and ion currents flowing onto the grain surface or to equilibrium charging processes [25–27]. It has been shown recently that grain charge fluctuations typically give rise to damping of waves that would otherwise propagate as normal modes [28,30].

Although a large body of literature has focused attention on collective effects in unmagnetized and magnetized dusty plasmas with a magnetohydrodynamics (MHD) description [21–23,31], consideration of thermal kinetic effects is also of great importance. It is anticipated that the presence of external magnetic fields with inclusion of thermal kinetic effects can open a number of additional or new modes or can change or modify the existing plasma modes in a dusty plasma. Since the mass of a dust grain can be many orders higher than that of the ions, the finite-Larmor-radius (FLR) effects must become significant in a hot magnetized dusty plasma. We show in this paper that the FLR effects of the massive and mobile charged dust grains can give rise to a very low-frequency (lower than the dust cyclotron frequency) electrostatic mode in the dusty plasma, which exists only in the presence of an external magnetic field and the mode disappears when the external magnetic field vanishes. This low-frequency electrostatic mode has no counterpart in the usual cold-fluid MHD description of the magnetized dusty plasma.

We consider a fully ionized plasma consisting of positive ions, electrons, and negatively charged dust grains. Although the size, the mass, and the charge of the dust grains vary from one grain to another, we assume that the dust grains all have uniform size and negative charge, and that they neither break up nor coalesce [21]. When the grain size is much smaller than the wavelength of perturbations and the interparticle distance, then the dust grains can be treated as negatively charged point masses (like negative ions). In the equilibrium, the plasma is quasineutral and the conservation of the particle number density must always hold. Thus

$$Z_e n_{e0} + Z_d n_{d0} = Z_i n_{i0}, \tag{1}$$

where $Z_e = 1$ and Z_i and Z_d refer to the charge states of ions and dust particles; n_{j0} is the unperturbed particle number density. We assume constant charge on the dust particles and thereby neglect any damping of the modes that may arise because of grain charge fluctuations. Such an approximation can be justified for those dusty plasmas that have fairly small dust particles with weak charging [21].

In this paper, we study the low-frequency modes in a hot and uniformly magnetized dusty plasma by considering dust dynamics with Vlasov-gyrokinetic theory. In particular, we investigate the low-frequency electrostatic modes propagating almost perpendicular as well as almost parallel to the external magnetic field and having frequency below the dust-cyclotron frequency. Let us now consider a very low-frequency electrostatic mode $(\omega, \mathbf{k})$ propagating obliquely to an external magnetic field $\mathbf{B}_s \| \hat{z}$. We assume that the wave vector $\mathbf{k}$ of the mode lies on the xz plane in a three-dimensional Cartesian coordinate system. The dispersion relation for this kind of electrostatic low-frequency mode can be obtained by considering the gyrokinetic theory of the multiple-species dusty plasma. The dispersion relation of this ultra-low-frequency electrostatic mode is obtained from [32]

$$\epsilon(\omega, \mathbf{k}) = 1 + \sum_{\alpha = e,i,d} \left(\frac{1}{k^2 \lambda_{D\alpha}^2} \right) [1 + \Gamma_{0\alpha} \zeta_\alpha Z(\zeta_\alpha)] \equiv 0, \tag{2}$$

1063-651X/96/54(3)/2232(4)/$10.00

where $\alpha=e,i,d$ represents plasma species, $\lambda_{D\alpha}^2=T_\alpha/4\pi Z_\alpha^2 e^2 n_{\alpha 0}$, $\zeta_\alpha=\omega/k_\parallel V_{\mathrm{th}\alpha}$, $\Gamma_{0\alpha}=I_0(b_\alpha)\exp(-b_\alpha)$, $b_\alpha=k_\perp^2 V_{th\alpha}^2/2\omega_{c\alpha}^2$; I_0 is the zero-order modified Bessel function with argument b_α and Z is the plasma dispersion function with argument ζ_α; $V_{\mathrm{th}\alpha}=(2T_\alpha/m_\alpha)^{1/2}$ and $\omega_{c\alpha}=Z_\alpha eB_s/m_\alpha c$, are, respectively, the thermal velocity and cyclotron frequency of particles of species α; m_α is the mass of a particle, and T_α is the temperature of species α; e is the magnitude of electron charge and c is the velocity of light in a vacuum.

We now solve Eq. (2) explicitly for the dispersion relation for low-frequency electrostatic waves for the following two conditions.

A. Quasiperpendicular low-frequency electrostatic dust mode

In this case, we assume that the mode $(\omega,\mathbf{k})$ under consideration satisfies the following conditions:

$$\frac{k_\perp V_{thd}}{\omega_{cd}} \gg 1 \gg \frac{k_\perp V_{thi}}{\omega_{ci}} \gg \frac{k_\perp V_{the}}{\omega_{ce}}, \tag{3}$$

$$\omega_{ce} \gg \omega_{ci} \gg \omega_{cd} \gg \omega, \tag{4}$$

$$k_\parallel V_{the} \gg k_\parallel V_{thi} \gg \omega \gg k_\parallel V_{thd}. \tag{5}$$

To satisfy these conditions simultaneously, we must have $k_\perp \gg k_\parallel$. This means that the mode should be quasiperpendicular. It is important to note that condition (4) is the essential criterion for the derivation of the dielectric response function, Eq. (2) for low-frequency electrostatic waves [32]. Using conditions (3)–(5), the dispersion relation, Eq. (2), for the mode under consideration reduces to

$$\omega^2=\left(\frac{\omega_{cd}}{\sqrt{\pi}k_\perp V_{thd}}\right) \times \frac{Z_d(\delta-1)k_\parallel^2 c_d^2}{1+Z_i^2(T_e/T_i)(n_{i0}/n_{e0})+Z_d^2(T_e/T_d)(n_{d0}/n_{e0})}, \tag{6}$$

where $\delta\equiv Z_i n_{i0}/n_{e0}>1$ and $c_d\equiv(T_e/m_d)^{1/2}$. The normalized Landau damping rate of this low-frequency mode is obtained as

$$\frac{\gamma_L}{\omega}=-\left(\frac{\pi\omega^3 k_\perp}{\omega_{cd}k_\parallel^3 V_{thd}^2}\right)\left[\left(\frac{\omega_{cd}}{\sqrt{\pi}k_\perp V_{thd}}\right)\exp(-\omega^2/k_\parallel^2 V_{thd}^2) + \left(\frac{Z_i}{Z_d}\right)^2\left(\frac{T_d}{T_i}\right)^{3/2}\left(\frac{m_i}{m_d}\right)^{1/2}\left(\frac{n_{i0}}{n_{d0}}\right)\exp(-\omega^2/k_\parallel^2 V_{thi}^2)\right]. \tag{7}$$

The electrostatic mode, given by Eq. (6) is an obliquely propagating low-frequency (lower than the dust cyclotron frequency) dust mode. It does not have a counterpart in the cold-fluid MHD description of the dusty plasma. This mode in the hot magnetized dusty plasma appears to be entirely due to the inequalities of charge and number densities of electrons, ions, and dust particles and the FLR thermal kinetic effects, as can be seen from Eq. (6). It is noted that the dispersion relation of the low-frequency electrostatic mode contains all the parameters of the dust component of the multiple-species dusty plasma. It is interesting to note from the dispersion relation, Eq. (6), that the frequency ω of this low-frequency mode becomes zero either when there is no external magnetic field ($B_s=0$) or when the mode is assumed to propagate exactly along the perpendicular direction ($k_\parallel=0$). Since the parallel component of the wave vector of the mode is very small compared to the perpendicular component ($k_\parallel \ll k_\perp$), the Landau damping rate of the mode is negligibly small, as is evident from the analytic expression for the damping, Eq. (7).

B. Slow-dust sound wave

We now assume a mode $(\omega,\mathbf{k})$ which satisfies the following conditions:

$$1 \gg \frac{k_\perp V_{thd}}{\omega_{cd}} \gg \frac{k_\perp V_{thi}}{\omega_{ci}} \gg \frac{k_\perp V_{the}}{\omega_{ce}}, \tag{8}$$

$$\omega_{ce} \gg \omega_{ci} \gg \omega_{cd} \gg \omega, \tag{9}$$

$$k_\parallel V_{the} \gg k_\parallel V_{thi} \gg \omega \gg k_\parallel V_{thd}. \tag{10}$$

Utilizing these conditions, Eq. (2) gives the following dispersion relation for the low-frequency mode $(\omega,\mathbf{k})$ in the hot magnetized dusty plasma:

$$\omega^2=\frac{Z_d(\delta-1)k_\parallel^2 c_d^2}{1+Z_i^2(T_e/T_i)(n_{i0}/n_{e0})+k_\parallel^2\lambda_{De}^2+\{\lambda_{De}^2+Z_d(\delta-1)\rho_d^2\}k_\perp^2}, \tag{11}$$

where $\rho_d \equiv c_d/\omega_{cd}$. The normalized Landau damping rate of this low-frequency mode is obtained as

$$\frac{\lambda_L}{\omega} = -\sqrt{\pi}\left(\frac{\omega}{k_\parallel V_{thd}}\right)^3\left[\exp\left\{-\left(\frac{\omega}{k_\parallel V_{thd}}\right)^2\right\} + \left(\frac{Z_i}{Z_d}\right)^2\left(\frac{T_d}{T_i}\right)^{3/2}\left(\frac{m_i}{m_d}\right)^{1/2}\left(\frac{n_{i0}}{n_{d0}}\right)\exp\left\{-\left(\frac{\omega}{k_\parallel V_{thi}}\right)^2\right\}\right]. \tag{12}$$

Equation (11) is the dispersion relation of a very low-frequency (lower than the dust cyclotron frequency) dust mode in the hot magnetized dusty plasma. The counterpart of this mode in a two-component electron-ion plasma is the slow-ion sound wave. Actually, the MHD version of this mode was first predicted by Rao, Shukla, and Yu [21], in an unmagnetized dusty plasma and the mode is known as dust acoustic wave. The mode described by Eq. (11) may be called slow-dust sound wave in the hot magnetized dusty plasma. As we see from Eq. (11), due to the FLR thermal kinetic effects, the mode acquires a finite perpendicular group velocity, i.e., the mode can now propagate across the external magnetic field and this can lead to cross-field energy transport, which is absent in the usual description of the unmagnetized cold dusty plasma [21]. In the $k_\perp \to 0$ limit, one recovers the MHD result of Rao, Shukla, and Yu, i.e., the so-called dust acoustic wave. We also notice from the expression for the Landau damping rate, Eq. (12), which is finite for this dust mode, the damping is due to both ions and dust grains.

In summary, we have presented a gyrokinetic study of low-frequency electrostatic modes in a hot magnetized dusty plasma. It is found that the inequalities of charge and number densities of the different plasma species and the FLR thermal kinetic effects of the charged and massive dust grains introduce the existence of electrostatic eigenmodes in a three-component uniform hot magnetized dusty plasma. In Case A, a quasiperpendicular low-frequency dust mode is found to exist due only to the FLR thermal kinetic effects of the massive, heavily charged and mobile dust grains and it does not have a counterpart in the cold-fluid MHD description of the dusty plasma. It is interesting to note that this particular low-frequency mode disappears either when the external magnetic field vanishes or when the mode is assumed to propagate along a direction exactly perpendicular to the external magnetic field. We have also studied the existence of a low-frequency dust mode, which may be called slow-dust sound wave (Case B) in the hot magnetized dusty plasma. This mode is the kinetic version of the so-called dust acoustic wave predicted by Rao, Shukla, and Yu [21], in an unmagnetized dusty plasma. The FLR effects have been included in the gyrokinetic theory to derive the dispersion relation of this mode. Due to the FLR effects, the slow-dust sound wave acquires a finite perpendicular group velocity. The expression for the Landau damping rate, which is finite for this mode, has also been determined.

In the present investigation, the dust particles were taken as a third component of the plasma having constant charge and mass. For simplicity, it was assumed that the charge on the dust particles is not affected by the wave. The variation of the dust charges can lead to additional damping [28–30] apart from the collisionless Landau damping of the modes derived in the present text. The charge fluctuation on the dust particles may be the dominant damping mechanism for the low-frequency modes, except for very long and short wavelengths, and the effect is maximum when the wave frequency is equal to the charging frequency of the dust [29]. Furthermore, the charging effects can lead also to renormalization of the electron plasma frequency, namely, to its decreasing [27]. Fluctuation of grain charge can be negligible as long as the grain thermal speed is much less than the electron or ion thermal speed.

The low-frequency modes studied here can have many applications in space and astrophysics [33–35] as well as in laboratory experiments for dust-particulate attraction during the formation of the dust crystals and dust coagulation [13–20]. It has been theoretically demonstrated recently that attraction between dust particulates is possible in a dusty plasma only due to the presence of low-frequency electrostatic dust modes [36,37]. The attraction between dust particulates can properly explain the formation of Coulomb dust crystals and the structure formations in astrophysical situations. Experimental observations [15,38] also indicate that prior to Coulomb crystallization, there appear low-frequency electrostatic fluctuations. It is important to mention here that extensive investigations have been done in this direction on the direct observation of macroscopic Coulomb crystal formation in dusty plasma environments both theoretically and experimentally in recent years [13–20]. The low-frequency dust modes studied here will be helpful in explaining the experiments of Coulomb dust crystallization in a realistic situation where an external magnetic field might be necessary for experimental purposes. Furthermore, these low-frequency dust modes and their consequent influence on dust coagulation may play an important role in the interstellar regions where the star-formation processes take place [39,40]. Collective effects on Coulomb dust crystallization and parametric mode-coupling interactions through these low-frequency modes in dusty plasmas will be important fields of research in the future.

The author would like to thank Professor M. Salimullah for a number of useful discussions.

[1] D. A. Mendis and M. Rosenberg, Annu. Rev. Astron. Astrophys. **32**, 419 (1994).
[2] T. G. Northrop, Phys. Scr. **45**, 475 (1992); U. de Angelis, *ibid.* **45**, 465 (1992).
[3] C. K. Goertz, Rev. Geophys. **27**, 271 (1989).
[4] V. N. Tsytovich, G. E. Morfill, R. Bingham, and U. de Angelis, Comments Plasma Phys. Controlled Fusion **13**, 153 (1990); V. N. Tsytovich, *ibid.* **15**, 349 (1994).

[5] R. C. Hazelton, IEEE Trans. Plasma Sci. **22**, 91 (1994).
[6] G. S. Selwyn, J. E. Heidenreich, and K. L. Haller, Appl. Phys. Lett. **57**, 1876 (1990).
[7] J. E. Daugherty, P. K. Porteous, M. D. Kilogre, and D. V. Graves, J. Appl. Phys. **72**, 3934 (1991).
[8] R. N. Carlile, S. Gehe, J. F. O'Hanlon, and J. C. Stewart, Appl. Phys. Lett. **59**, 1167 (1991).
[9] D. P. Sheehan, M. Carillo, and W. Heidbrink, Rev. Sci. Instrum. **61**, 3871 (1990).
[10] A. Bouchoule, A. Plain, L. Boufendi, J. Ph. Bloudeau, and C. Laure, J. Appl. Phys. **70**, 1991 (1991).
[11] S. Hamaguchi and R. T. Farouki, Phys. Plasmas **1**, 2110 (1994).
[12] J.-X. Ma and M. Y. Yu, Phys. Plasmas **2**, 1343 (1995).
[13] H. Ikezi, Phys. Fluids **29**, 1764 (1986).
[14] H. Thomas, G. E. Morfill, V. Demmel, J. Goree, B. Feuerbacher, and D. Mohlmann, Phys. Rev. Lett. **73**, 652 (1994).
[15] J. H. Chu, J. B. Du, and Lin I, J. Phys. D **27**, 296 (1994).
[16] J. H. Chu and Lin I, Phys. Rev. Lett. **72**, 4009 (1994).
[17] Y. Hayashi and K. Tachibana, Jpn. J. Appl. Phys. **33**, L804 (1994).
[18] A. Melzer, T. Trottenberg, and A. Piel, Phys. Lett. A **191**, 301 (1994).
[19] L. Boufendi and A. Bouchoule, Plasma Sources Sci. Technol. **3**, 262 (1994).
[20] M. Shiratani, H. Kawasaki, T. Fukuzawa, H. Tsuruoka, T. Yoshioka, and Y. Watanabe, Appl. Phys. Lett. **65**, 1900 (1994).
[21] N. N. Rao, P. K. Shukla, and M. Y. Yu, Planet. Space Sci. **38**, 543 (1990); P. K. Shukla, Phys. Scr. **45**, 504 (1992).
[22] N. N. Rao, J. Plasma Phys. **49**, 375 (1993); Planet. Space Sci. **41**, 21 (1993); J. Plasma Phys. **53**, 317 (1995).
[23] N. D'Angelo, Planet. Space Sci. **38**, 1143 (1990); *ibid.* **41**, 469 (1993); A. Barkan, R. L. Marlino, and N. D'Angelo, Phys. Plasmas **2**, 3563 (1995).
[24] N. D'Angelo and B. Song, Planet. Space Sci. **38**, 1577 (1990); M. Rosenberg, *ibid.* **41**, 229 (1993)
[25] M. S. Barnes, J. H. Keller, J. C. Forster, J. A. O'Neill, and D. K. Coultas, Phys. Rev. Lett. **68**, 313 (1992).
[26] V. N. Tsytovich and O. Havnes, Comments Plasma Phys. Controlled Fusion **15**, 267 (1993).
[27] S. V. Vladimirov, Phys. Plasmas **1**, 2762 (1994).
[28] R. K. Varma, P. K. Shukla, and V. Krishan, Phys. Rev. E **47**, 3612 (1993).
[29] F. Melandso, T. K. Aslaksen, and O. Havnes, Planet. Space Sci. **41**, 312 (1993); J. Geophys. Res. **98**, 13315 (1993).
[30] N. N. Rao and P. K. Shukla, Planet. Space Sci. **42**, 221 (1994).
[31] F. Verheest, Space Sci. Rev. **68**, 109 (1994).
[32] Liu Chen, *Waves and Instabilities in Plasmas* (World Scientific, Singapore, 1987).
[33] R. Grard *et al.*, Nature **321**, 290 (1986).
[34] S. Klimov *et al.*, Nature **321**, 292 (1986).
[35] W. Pilipp, T. W. Hartquist, and O. Havnes, Astrophys. J. **314**, 341 (1987).
[36] M. Nambu, S. V. Vladimirov, and P. K. Shukla, Phys. Lett. A **203**, 40 (1995).
[37] S. V. Vladimirov and M. Nambu, Phys. Rev. E **52**, R2172 (1995).
[38] R. L. Merlino, A. Barkan, N. D'Angelo, W. Xu, and B. Song, in *Proceedings of the International Conference on Plasma Physics*, edited by P. H. Sakanaka and M. Tendler, AIP Conf. Proc. No. 345 (AIP, New York, 1995), pp. 295–302.
[39] F. H. Shu, F. C. Adams, and S. Lizano, in *Evolution of Interstellar Dust and Related Topics*, Proceedings of the International School of Physics ''Enrico Fermi,'' Course CI, Varenna, 1986, edited by A. Bonetti, J. M. Greenberg, and S. Aiellu (North-Holland, Amsterdam, 1989), p. 397.
[40] L. Mestel, in *Protostar and Planet II*, edited by D. C. Block and M. S. Mathews (University of Arizona Press, Tucson, AZ, 1985), p. 321.